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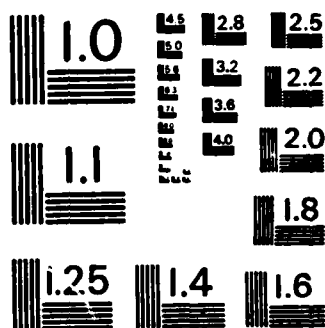
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CATEGORY ANALYSIS OF THE MARINE CORPS
COMBAT READINESS EVALUATION SYSTEM (MCCRES)

by

S. Zacks, W. H. Marlow and Zeev Barzily

Serial T-450 ✓
15 October 1981

The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

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CATEGORY ANALYSIS OF THE MARINE CORPS
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1. Introduction

The Marine Corps Combat Readiness Evaluation System (MCCRES) and various analyses have been discussed recently in a series of papers by Barzily, Catalogne and Marlow [2], Zacks and Marlow [7] and Barzily, Marlow, and Zacks [8]. In the present paper we conduct statistical analysis of MCCRES from the point of view of the ten categories introduced by Barzily [1]. As in the previous work we restrict our present attention to Volume II of MCCRES, which deals with infantry battalions and contains 800 different requirements. The category approach of Barzily classifies all requirements into the following ten categories.

1. REPORTING to higher levels of command
2. PREPARING for operation
3. COMMUNICATING
4. PERFORMING as Marines
5. DELIVERING supporting fire
6. PLANNING of operations
7. CONFORMING to doctrine

8. EXECUTING operations
9. PROVIDING combat service support
10. SUPERVISING required actions of individual Marines

In a given evaluation an infantry battalion is evaluated on a subset of the 800 requirements. That is, due to time, budget, and other constraints, not all 800 requirements are applicable. The battalion is given a score of "Yes" in case a requirement is fulfilled satisfactorily, and a score of "No" otherwise. The basic assumption in our category approach is that the probability of "Yes" for a given unit is the same for all the applicable requirements of the same category (see Appendix 1). These probabilities may change from one category to another. The vector of the ten probabilities, corresponding to the ten categories, is a parametric vector which characterizes the unit at that particular evaluation. Different units usually have different characterizing parametric vectors and the same unit may have different characterizing parametric vectors in different evaluations. In addition to the parametric vector of "Yes" probabilities, there are several other interesting statistical features of the data. The proportion of "Yes's" among the applicable requirements of different categories in the same evaluation are not independent. It is interesting to study the correlation structure between these categories. In the present study we present statistical models and develop procedures designed to estimate interesting model parameters. We base the analysis on the results of $N = 27$ evaluations. The statistical methods developed here have a wide range of possible applications for the analysis of categorical data in which a different number of independent Bernoulli trials are performed in each category, but the proportions of successes in different categories are dependent variables.

2. The Structure of the Data and the Statistical Model

Let N designate the number of evaluations, $N = 27$, and k designate the number of categories, $k = 10$. Let X_{ij} , $i=1, \dots, N$, $j=1, \dots, k$ designate the number of "Yes's" among the applicable requirements of the

j -th category in the i -th evaluation. Let M_{ij} , $i=1, \dots, N$, $j=1, \dots, k$ designate the corresponding number of applicable requirements. In Table 2.1 we present the values of X_{ij} and M_{ij} which have been compiled from MCCRES data files. All the analyses in the present paper are based on the data in this table.

Let \underline{X}_i ($i=1, \dots, N$) designate the vector of k X - values corresponding to evaluation i , and \underline{M}_i the vectors of corresponding M - values. The fundamental assumptions of the statistical model are:

A.1 $\underline{X}_1, \underline{X}_2, \dots, \underline{X}_N$ are independent random vectors;

A.2 The marginal distribution of X_{ij} is binomial,
 $B(M_{ij}, \theta_{ij})$, for all $i=1, \dots, N$, and $j=1, \dots, k$.
 θ_{ij} designates the probability of "Yes" for all
the requirements in evaluation i and category j .

In Appendix 1 we test a null hypothesis which is based on assumption A.2. This hypothesis is accepted by the data in such a way to provide substantial empirical verification.

The multivariate distributions of \underline{X}_i , $i=1, \dots, N$, are specified indirectly by considering first the following transformations. Let

$$Y_{ij} = 2 \sin^{-1} \left(\sqrt{\frac{X_{ij} + 1/2}{M_{ij} + 1}} \right), \quad \begin{matrix} i=1, \dots, N \\ j=1, \dots, k \end{matrix} \quad (2.1)$$

These transformations are known as variance stabilizing transformations for binomial random variables (see Johnson and Kotz [3; p. 65], who suggest the transformation $Y = 2 \sin^{-1} \sqrt{\hat{\theta}}$, where $\hat{\theta} = (x + 3/8)/(M + 3/4)$. The asymptotic properties of (2.1) and of this one are the same).

Table 2.1

Total number of satisfied and applicable requirements
by category and evaluation*

| EVAL/CAT. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------|-----|----|----|----|----|-----|----|-----|----|----|
| 1 | 62 | 44 | 40 | 14 | 55 | 100 | 50 | 53 | 24 | 13 |
| | 67 | 54 | 40 | 23 | 69 | 110 | 63 | 73 | 32 | 17 |
| 2 | 79 | 70 | 42 | 18 | 67 | 133 | 65 | 97 | 31 | 17 |
| | 80 | 76 | 44 | 24 | 72 | 140 | 73 | 100 | 37 | 20 |
| 3 | 41 | 19 | 26 | 12 | 45 | 38 | 44 | 47 | 15 | 12 |
| | 42 | 26 | 28 | 21 | 56 | 45 | 53 | 54 | 16 | 14 |
| 4 | 60 | 33 | 25 | 20 | 53 | 68 | 49 | 68 | 18 | 16 |
| | 62 | 36 | 31 | 24 | 55 | 71 | 54 | 72 | 19 | 16 |
| 5 | 35 | 34 | 35 | 21 | 52 | 93 | 48 | 46 | 18 | 12 |
| | 52 | 44 | 37 | 24 | 54 | 99 | 54 | 67 | 21 | 14 |
| 6 | 52 | 29 | 20 | 23 | 57 | 60 | 59 | 62 | 18 | 18 |
| | 55 | 29 | 29 | 24 | 63 | 63 | 61 | 67 | 18 | 18 |
| 7 | 44 | 41 | 32 | 15 | 63 | 90 | 47 | 50 | 30 | 13 |
| | 47 | 46 | 34 | 21 | 64 | 93 | 51 | 55 | 31 | 17 |
| 8 | 29 | 18 | 28 | 12 | 48 | 40 | 34 | 65 | 7 | 9 |
| | 60 | 48 | 32 | 22 | 63 | 81 | 57 | 87 | 14 | 15 |
| 9 | 64 | 41 | 36 | 25 | 64 | 78 | 44 | 93 | 16 | 18 |
| | 71 | 48 | 37 | 26 | 64 | 90 | 57 | 104 | 18 | 18 |
| 10 | 64 | 48 | 35 | 19 | 60 | 99 | 45 | 67 | 25 | 17 |
| | 70 | 55 | 38 | 24 | 67 | 119 | 61 | 77 | 30 | 17 |
| 11 | 15 | 4 | 23 | 18 | 56 | 15 | 30 | 17 | 13 | 11 |
| | 15 | 4 | 23 | 18 | 56 | 15 | 31 | 17 | 13 | 12 |
| 12 | 12 | 5 | 14 | 16 | 35 | 10 | 16 | 12 | 8 | 7 |
| | 14 | 8 | 17 | 16 | 54 | 16 | 30 | 18 | 10 | 10 |
| 13 | 11 | 11 | 5 | 6 | 6 | 15 | 8 | 36 | 2 | 5 |
| | 11 | 11 | 6 | 9 | 6 | 16 | 10 | 37 | 2 | 5 |
| 14 | 54 | 31 | 23 | 17 | 57 | 55 | 49 | 62 | 16 | 13 |
| | 54 | 35 | 27 | 25 | 59 | 62 | 57 | 70 | 17 | 18 |
| 15 | 41 | 21 | 13 | 16 | 4 | 36 | 33 | 44 | 13 | 11 |
| | 42 | 23 | 17 | 23 | 4 | 36 | 35 | 47 | 13 | 16 |
| 16 | 49 | 35 | 29 | 13 | 59 | 73 | 59 | 63 | 18 | 12 |
| | 54 | 38 | 32 | 23 | 59 | 74 | 63 | 70 | 18 | 16 |
| 17 | 44 | 29 | 18 | 16 | 9 | 52 | 44 | 54 | 18 | 13 |
| | 47 | 30 | 21 | 23 | 9 | 52 | 45 | 55 | 19 | 13 |
| 18 | 69 | 40 | 35 | 26 | 62 | 76 | 66 | 87 | 22 | 14 |
| | 69 | 41 | 36 | 28 | 63 | 78 | 68 | 90 | 22 | 16 |
| 19 | 78 | 43 | 38 | 27 | 64 | 90 | 60 | 92 | 19 | 17 |
| | 79 | 51 | 38 | 27 | 65 | 98 | 71 | 108 | 20 | 17 |
| 20 | 48 | 26 | 25 | 13 | 58 | 49 | 38 | 56 | 16 | 17 |
| | 54 | 31 | 29 | 24 | 61 | 61 | 56 | 67 | 18 | 18 |
| 21 | 18 | 5 | 14 | 4 | 41 | 14 | 22 | 12 | 12 | 9 |
| | 24 | 11 | 21 | 17 | 56 | 22 | 36 | 28 | 14 | 15 |
| 22 | 71 | 35 | 29 | 21 | 52 | 85 | 45 | 85 | 24 | 19 |
| | 73 | 37 | 34 | 25 | 66 | 92 | 54 | 90 | 24 | 20 |
| 23 | 71 | 41 | 33 | 26 | 63 | 87 | 63 | 93 | 21 | 15 |
| | 73 | 41 | 36 | 28 | 64 | 88 | 68 | 97 | 23 | 16 |
| 24 | 47 | 30 | 28 | 22 | 55 | 61 | 52 | 49 | 13 | 16 |
| | 49 | 31 | 30 | 25 | 57 | 63 | 57 | 58 | 15 | 17 |
| 25 | 44 | 32 | 29 | 16 | 31 | 56 | 50 | 47 | 15 | 16 |
| | 48 | 34 | 32 | 25 | 61 | 64 | 58 | 58 | 16 | 18 |
| 26 | 34 | 29 | 24 | 21 | 52 | 48 | 32 | 42 | 15 | 14 |
| | 37 | 32 | 27 | 23 | 58 | 56 | 40 | 54 | 15 | 15 |
| 27 | 100 | 58 | 32 | 27 | 69 | 121 | 75 | 136 | 26 | 18 |
| | 101 | 58 | 38 | 29 | 71 | 122 | 78 | 138 | 28 | 19 |

*The upper number in each cell represents the total number of satisfied requirements; the lower number represents the total number of applicable requirements.

For large values of M_{ij} the random variables Y_{ij} are distributed approximately normally, with mean $\eta_{ij} = 2 \sin^{-1} \sqrt{\theta_{ij}}$ and variance $1/M_{ij}$. Let $\underline{Y}_i = (Y_{i1}, \dots, Y_{ik})'$ be the k -variate vector of the transformed X -values of the i -th evaluations. According to assumption A.1, $\underline{Y}_1, \dots, \underline{Y}_N$ are independent random vectors. We introduce here a third assumption concerning the multivariate distribution of \underline{Y}_i ($i=1, \dots, N$).

A.3 The random vectors \underline{Y}_i have multivariate normal distributions with means $\underline{\eta}_i = (\eta_{i1}, \dots, \eta_{ik})'$ and covariance matrices $\Phi_i = (\Phi_{i.jj'})$,

where

$$\Phi_{i.jj'} = \begin{cases} \frac{1}{M_{ij}} & , \quad \text{if } j=j' \\ \rho_{jj'} / \sqrt{M_{ij} M_{ij'}} & , \quad \text{if } j \neq j' \end{cases} \quad (2.2)$$

$\rho_{jj'}$ is the correlation between Y_{ij} and $Y_{ij'}$.

It is assumed that the correlations $\rho_{jj'}$ are the same for all evaluations. In Appendix 2 we show that $\rho_{jj'}$ are asymptotically equivalent to the correlations between the proportions of Yes's, $\hat{\theta}_{ij} = X_{ij}/M_{ij}$.

3. The Bayesian Framework

The Bayesian model assumes that the characterizing parametric vectors $\underline{\theta}_i$ of the different evaluations vary according to a (prior) distribution of all possible evaluations. This is a model of a superpopulation which makes it possible to relate the information obtained from all evaluations in a consistent manner. Following the model developed in the previous section, we present here the Bayesian model in terms of the distributions of \underline{Y}_i ($i=1, \dots, N$).

We assume that the mean vectors $\eta_{i\cdot}$ of the distributions of $Y_i (i=1, \dots, N)$ have a prior normal distribution with mean vector $\bar{\eta}$ and diagonal covariance matrix (D) . The overall distribution of Y_{ij} , where η_{ij} are treated as random variables, is called a predictive distribution. According to the above assumption, the predictive distribution of each Y_i is normal with the same mean, $\bar{\eta}$, and covariance matrix $\frac{1}{N} + (D)$ (see Zacks [6; p. 296]). The predictive marginal distributions of the components Y_{ij} are normal with means $\bar{\eta}_j (j=1, \dots, k)$ and variances

$\tau_{ij}^2 = \frac{1}{M_{ij}} + D_j^2 (i=1, \dots, N, j=1, \dots, k)$. Since we assumed that the correlations $\rho_{jj'}$ are the same for all evaluations, we will treat the vectors $\bar{\eta}$, and the matrices (D) and $(R) = (\rho_{jj'}, j, j' = 1, \dots, k)$ as unknown but fixed parameters. We remark that the predictive marginal distributions of Y_{ij} resemble the distributions of the observations in Model II of Analysis of Variance (see Zacks [6; p. 79]), in which the components of variance are $1/M_{ij}$ (known) and D_j (unknown). We conclude this section by specifying the posterior distributions of η_{ij} given Y_{ij} . According to the Bayesian model, the posterior distribution of η_{ij} given Y_{ij} is normal with mean

$$E \{ \eta_{ij} | Y_{ij} \} = (\bar{\eta}_j + D_j M_{ij} Y_{ij}) / (1 + D_j M_{ij}), \quad (3.1)$$

and variance

$$V \{ \eta_{ij} | Y_{ij} \} = D_j / (1 + M_{ij} D_j) \quad (3.2)$$

Notice that the Bayes estimator of η_{ij} for a squared-error loss is given by (3.1), with a Bayes risk given by (3.2). If $\bar{\eta}_j$ and D_j are unspecified, one can estimate them from the data, to obtain empirical Bayes estimators of the characterizing parameters (see Zacks [6; p. 321]).

4. Bayesian Confidence Intervals for the Characterizing Parameters

Bayesian confidence intervals, at level $1-\alpha$, for specified parameters, are defined as the intervals in the parameterspace whose posterior coverage probabilities, given the sample data, are at least $1-\alpha$. According to the model of Section 3, the posterior coverage probability of the intervals

$$\frac{\bar{\eta}_j + D_j M_{ij} Y_{ij}}{1 + I_j M_{ij}} \pm Z_{1-\alpha/2} \left[\frac{D_j}{1 + M_{ij} D_j} \right]^{1/2} \quad (4.1)$$

is $(1-\alpha)$, where Z_γ is the γ -th fractile of the standard normal distribution. We determine for each category empirical estimates of the prior parameters $\bar{\eta}_j$ and D_j and, by substituting these estimates in (4.1), we obtain empirical Bayes confidence intervals for η_{ij} ($i=1, \dots, N$). The substitution of estimates of $\bar{\eta}_j$ and D_j for the correct values will generally effect the coverage probabilities. The estimates described below are consistent, and we expect that, for large values of N , the actual coverage will be close to the nominal one. In order to provide conservative intervals, we have used in each category, the Bonferroni method of simultaneous confidence intervals, for all the $N = 27$ evaluations (see Miller [4]). For this purpose we replaced α in (4.1) by $\alpha' = \alpha/N$.

Applying the transformation

$$\theta = \sin^2(\eta/2), \quad (4.2)$$

which is strictly increasing for $0 \leq \eta \leq \pi$, we can obtain from the η -confidence intervals θ -confidence intervals. These intervals, for the ten categories and twenty seven evaluations with $\alpha = .05$, are presented in Table 4.1.

Table 4.1

CONFIDENCE INTERVALS FOR CAT. C1-C5

| EVAL | 1 | | 2 | | 3 | | 4 | | 5 | |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | L | U | L | U | L | U | L | U | L | U |
| 1 | 0.802 | 0.987 | 0.648 | 0.942 | 0.849 | 0.999 | 0.366 | 0.882 | 0.661 | 0.926 |
| 2 | 0.902 | 1.000 | 0.794 | 0.982 | 0.802 | 0.988 | 0.481 | 0.942 | 0.809 | 0.988 |
| 3 | 0.834 | 1.000 | 0.519 | 0.942 | 0.751 | 0.984 | 0.333 | 0.872 | 0.655 | 0.940 |
| 4 | 0.853 | 0.999 | 0.723 | 0.991 | 0.683 | 0.956 | 0.556 | 0.972 | 0.834 | 0.999 |
| 5 | 0.548 | 0.881 | 0.592 | 0.931 | 0.782 | 0.987 | 0.598 | 0.984 | 0.832 | 0.999 |
| 6 | 0.814 | 0.995 | 0.820 | 1.000 | 0.625 | 0.931 | 0.699 | 1.000 | 0.769 | 0.980 |
| 7 | 0.791 | 0.995 | 0.717 | 0.980 | 0.772 | 0.986 | 0.437 | 0.935 | 0.880 | 1.000 |
| 8 | 0.397 | 0.749 | 0.269 | 0.668 | 0.723 | 0.972 | 0.318 | 0.855 | 0.622 | 0.912 |
| 9 | 0.778 | 0.977 | 0.680 | 0.964 | 0.807 | 0.993 | 0.717 | 1.000 | 0.913 | 1.000 |
| 10 | 0.792 | 0.983 | 0.711 | 0.968 | 0.763 | 0.981 | 0.517 | 0.958 | 0.763 | 0.975 |
| 11 | 0.755 | 1.000 | 0.509 | 1.000 | 0.794 | 0.997 | 0.714 | 1.000 | 0.902 | 1.000 |
| 12 | 0.634 | 0.997 | 0.407 | 0.982 | 0.679 | 0.973 | 0.688 | 1.000 | 0.514 | 0.858 |
| 13 | 0.716 | 1.000 | 0.659 | 1.000 | 0.668 | 0.986 | 0.328 | 0.969 | 0.621 | 1.000 |
| 14 | 0.900 | 1.000 | 0.685 | 0.984 | 0.704 | 0.969 | 0.427 | 0.909 | 0.843 | 0.999 |
| 15 | 0.834 | 1.000 | 0.669 | 0.996 | 0.658 | 0.966 | 0.432 | 0.922 | 0.582 | 1.000 |
| 16 | 0.768 | 0.985 | 0.734 | 0.992 | 0.743 | 0.979 | 0.335 | 0.860 | 0.906 | 1.000 |
| 17 | 0.791 | 0.995 | 0.767 | 1.000 | 0.698 | 0.975 | 0.432 | 0.922 | 0.669 | 1.000 |
| 18 | 0.920 | 1.000 | 0.817 | 1.000 | 0.804 | 0.993 | 0.682 | 0.995 | 0.878 | 1.000 |
| 19 | 0.901 | 1.000 | 0.673 | 0.957 | 0.844 | 0.999 | 0.793 | 1.000 | 0.881 | 1.000 |
| 20 | 0.747 | 0.979 | 0.626 | 0.972 | 0.712 | 0.970 | 0.321 | 0.843 | 0.824 | 0.996 |
| 21 | 0.586 | 0.965 | 0.331 | 0.929 | 0.621 | 0.943 | 0.128 | 0.711 | 0.589 | 0.903 |
| 22 | 0.871 | 0.999 | 0.762 | 0.997 | 0.712 | 0.965 | 0.568 | 0.973 | 0.649 | 0.924 |
| 23 | 0.871 | 0.999 | 0.865 | 1.000 | 0.757 | 0.980 | 0.682 | 0.995 | 0.880 | 1.000 |
| 24 | 0.823 | 0.998 | 0.772 | 1.000 | 0.758 | 0.985 | 0.609 | 0.985 | 0.839 | 0.999 |
| 25 | 0.770 | 0.990 | 0.747 | 0.997 | 0.743 | 0.979 | 0.396 | 0.890 | 0.401 | 0.754 |
| 26 | 0.753 | 0.994 | 0.699 | 0.991 | 0.724 | 0.977 | 0.634 | 0.994 | 0.754 | 0.979 |
| 27 | 0.921 | 1.000 | 0.901 | 1.000 | 0.709 | 0.960 | 0.691 | 0.995 | 0.866 | 0.999 |

CONFIDENCE INTERVALS FOR CAT. C6-C10

| | | | | | | | | | | |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | 0.807 | 0.971 | 0.642 | 0.921 | 0.586 | 0.876 | 0.771 | 0.942 | 0.611 | 0.962 |
| 2 | 0.872 | 0.988 | 0.752 | 0.966 | 0.881 | 0.997 | 0.789 | 0.950 | 0.657 | 0.973 |
| 3 | 0.675 | 0.966 | 0.665 | 0.946 | 0.710 | 0.967 | 0.804 | 0.965 | 0.644 | 0.979 |
| 4 | 0.844 | 0.996 | 0.747 | 0.979 | 0.821 | 0.991 | 0.808 | 0.966 | 0.736 | 0.997 |
| 5 | 0.840 | 0.987 | 0.727 | 0.971 | 0.545 | 0.857 | 0.793 | 0.958 | 0.644 | 0.979 |
| 6 | 0.827 | 0.996 | 0.835 | 0.997 | 0.791 | 0.985 | 0.819 | 0.971 | 0.749 | 0.997 |
| 7 | 0.877 | 0.997 | 0.759 | 0.984 | 0.755 | 0.982 | 0.825 | 0.970 | 0.611 | 0.962 |
| 8 | 0.385 | 0.704 | 0.465 | 0.812 | 0.616 | 0.880 | 0.759 | 0.944 | 0.549 | 0.940 |
| 9 | 0.746 | 0.954 | 0.615 | 0.914 | 0.783 | 0.963 | 0.797 | 0.961 | 0.749 | 0.997 |
| 10 | 0.723 | 0.924 | 0.588 | 0.892 | 0.736 | 0.957 | 0.789 | 0.952 | 0.743 | 0.997 |
| 11 | 0.733 | 1.000 | 0.765 | 0.999 | 0.737 | 1.000 | 0.811 | 0.970 | 0.662 | 0.987 |
| 12 | 0.451 | 0.950 | 0.406 | 0.843 | 0.470 | 0.945 | 0.788 | 0.960 | 0.586 | 0.968 |
| 13 | 0.671 | 1.000 | 0.523 | 0.986 | 0.801 | 1.000 | 0.794 | 0.966 | 0.640 | 0.993 |
| 14 | 0.744 | 0.974 | 0.700 | 0.957 | 0.746 | 0.967 | 0.805 | 0.965 | 0.594 | 0.952 |
| 15 | 0.856 | 1.000 | 0.747 | 0.994 | 0.775 | 0.993 | 0.811 | 0.970 | 0.580 | 0.951 |
| 16 | 0.894 | 1.000 | 0.796 | 0.987 | 0.763 | 0.974 | 0.819 | 0.971 | 0.604 | 0.961 |
| 17 | 0.895 | 1.000 | 0.823 | 0.999 | 0.856 | 1.000 | 0.808 | 0.966 | 0.714 | 0.996 |
| 18 | 0.876 | 0.999 | 0.850 | 0.997 | 0.869 | 0.996 | 0.826 | 0.973 | 0.658 | 0.980 |
| 19 | 0.812 | 0.978 | 0.701 | 0.944 | 0.737 | 0.937 | 0.809 | 0.966 | 0.743 | 0.997 |
| 20 | 0.654 | 0.934 | 0.532 | 0.863 | 0.689 | 0.944 | 0.797 | 0.961 | 0.705 | 0.989 |
| 21 | 0.464 | 0.928 | 0.463 | 0.863 | 0.323 | 0.797 | 0.792 | 0.961 | 0.549 | 0.940 |
| 22 | 0.815 | 0.982 | 0.670 | 0.947 | 0.837 | 0.989 | 0.829 | 0.974 | 0.717 | 0.990 |
| 23 | 0.909 | 1.000 | 0.789 | 0.983 | 0.862 | 0.993 | 0.804 | 0.963 | 0.691 | 0.989 |
| 24 | 0.851 | 0.999 | 0.758 | 0.980 | 0.688 | 0.954 | 0.793 | 0.961 | 0.698 | 0.989 |
| 25 | 0.733 | 0.968 | 0.704 | 0.958 | 0.653 | 0.937 | 0.804 | 0.965 | 0.672 | 0.981 |
| 26 | 0.704 | 0.964 | 0.617 | 0.943 | 0.617 | 0.923 | 0.814 | 0.970 | 0.684 | 0.988 |
| 27 | 0.933 | 1.000 | 0.847 | 0.994 | 0.924 | 0.999 | 0.810 | 0.964 | 0.711 | 0.990 |

The empirical estimates for $\bar{\eta}_j$ and D_j applied here are:

$$\hat{\eta}_{j,N} = \frac{1}{N} \sum_{i=1}^N Y_{ij}^2, \quad (4.3)$$

and

$$\hat{D}_{j,N} = \left[\frac{1}{N} \sum_{i=1}^N Y_{ij}^2 - \hat{\eta}_{j,N}^2 - H_{j,N}^{-1} \right]^+, \quad (4.4)$$

where $[a]^+ = \max(a, 0)$, and $H_{j,N}$ is the harmonic mean of M_{ij} , i.e., $H_{j,N}^{-1} = \frac{1}{N} \sum_{i=1}^N \frac{1}{M_{ij}}$. Notice that $\hat{\eta}_{j,N}$ is a strongly consistent estimator of $\bar{\eta}_j$ (approaches $\bar{\eta}_j$ a.s. as $N \rightarrow \infty$). Assuming that $H_{j,N}$ converges to a limit as $N \rightarrow \infty$, $\hat{D}_{j,N}$ converges to D_j as N grows. The values of $\hat{\eta}_{j,N}$ and $D_{j,N}$ obtained in this manner are given in Table 4.2.

Table 4.2
Empirical Estimates
of $\bar{\eta}_j$ and D_j

| j | $\bar{\eta}_{j,N}$ | $\hat{D}_{j,N}$ |
|-----|--------------------|-----------------|
| 1 | 2.5851 | 0.0755 |
| 2 | 2.4125 | 0.1058 |
| 3 | 2.4494 | 0.0297 |
| 4 | 2.1691 | 0.1464 |
| 5 | 2.5379 | 0.0871 |
| 6 | 2.5185 | 0.0994 |
| 7 | 2.3468 | 0.0751 |
| 8 | 2.4176 | 0.0917 |
| 9 | 2.4886 | 0.0086 |
| 10 | 2.3883 | 0.0388 |

5. Estimating the Correlations

The marginal bivariate predictive distribution of Y_{ij} and $Y_{ij'}$, for $j \neq j'$ is normal with p.d.f.

$$f(Y_{ij}, Y_{ij'}; \bar{\eta}_j, \bar{\eta}_{j'}, \rho_{jj'}, D_j, D_{j'}) =$$

$$\frac{1}{2\pi} \cdot \left(\frac{M_{ij}}{1 + D_j M_{ij}} \right)^{1/2} \left(\frac{M_{ij'}}{1 + D_{j'} M_{ij'}} \right)^{1/2} \cdot \frac{1}{\sqrt{1 - \rho_{jj'}^2}} \cdot$$

$$\exp \left\{ - \frac{1}{2(1 - \rho_{jj'}^2)} \left[\frac{M_{ij}}{1 + M_{ij} D_j} (Y_{ij} - \bar{\eta}_j)^2 + \frac{M_{ij'}}{1 + M_{ij'} D_{j'}} (Y_{ij'} - \bar{\eta}_{j'})^2 \right. \right.$$

$$\left. \left. - 2 \rho_{jj'} \frac{(Y_{ij} - \bar{\eta}_j) \sqrt{M_{ij}}}{\sqrt{1 + M_{ij} D_j}} \cdot \frac{(Y_{ij'} - \bar{\eta}_{j'}) \sqrt{M_{ij'}}}{\sqrt{1 + M_{ij'} D_{j'}}} \right] \right\} \quad (5.1)$$

Define the weight functions

$$w_{ij} = \frac{M_{ij}}{1 + M_{ij} D_j}, \quad i=1, \dots, N, \quad j=1, \dots, K. \quad (5.2)$$

If the prior means and variances, $\bar{\eta}_j, D_j$ ($j=1, \dots, k$) are known, then, the predictive quasi-likelihood function of $\rho_{jj'}$, given the sample data is

$$L(\rho_{jj'} | Y_{ij}, Y_{ij'}, i=1, \dots, N) =$$

$$(1 - \rho_{jj'}^2)^{N/2} \exp \left\{ - \frac{1}{2(1 - \rho_{jj'}^2)} \left[\sum_{i=1}^N w_{ij} (Y_{ij} - \bar{\eta}_j)^2 + \right. \right.$$

$$\left. \left. \sum_{i=1}^N w_{ij'} (Y_{ij'} - \bar{\eta}_{j'})^2 - 2 \rho_{jj'} \sum_{i=1}^N \sqrt{w_{ij} w_{ij'}} (Y_{ij} - \bar{\eta}_j) (Y_{ij'} - \bar{\eta}_{j'}) \right] \right\} \quad (5.3)$$

(5.3) is called a 'quasi-likelihood' since the correct likelihood function is based on the k -variate distributions of Y_1, \dots, Y_N .

In practice, $\bar{\eta}_j$ and D_j are unknown, we will therefore use the quasi-likelihood function of ρ_{jj} , by substituting in (5.3) estimates of $\bar{\eta}_j$ and D_j . Thus, let

$$S_j^2 = \sum_{i=1}^N \hat{W}_{ij} (Y_{ij} - \hat{\eta}_j)^2, \quad j=1, \dots, k$$

and

(5.4)

$$P_{jj'} = \sum_{i=1}^N \sqrt{\hat{W}_{ij} \hat{W}_{ij'}} (Y_{ij} - \hat{\eta}_j) (Y_{ij'} - \hat{\eta}_{j'}), \quad j \neq j'$$

where

$$\hat{\eta}_j = \frac{\sum_{i=1}^N \hat{W}_{ij} Y_{ij}}{\sum_{i=1}^N \hat{W}_{ij}}, \quad (5.5)$$

and

$$\hat{W}_{ij} = M_{ij} / (1 + M_{ij} \hat{D}_{j,N}) \quad (5.6)$$

The estimator $\hat{\rho}_{j,j'}$, which maximizes the log-quasi-likelihood

$$\ell(\rho_{jj'}) = \frac{N}{2} \log (1 - \rho_{jj'}^2) - \frac{1}{2(1 - \rho_{jj'}^2)} \left[S_j^2 + S_{j'}^2 - 2\rho_{jj'} P_{jj'} \right], \quad (5.7)$$

is called quasi-maximum likelihood (q.m.l.) estimator of $\rho_{jj'}$. The derivative of $\ell(\rho)$ with respect to ρ is

$$\frac{\partial \ell(\rho)}{\partial \rho} = N \frac{\rho}{1 - \rho^2} - \frac{\rho(S_j^2 + S_{j'}^2) - P_{jj'}(1 + \rho^2)}{(1 - \rho^2)^2}. \quad (5.8)$$

Accordingly, the q.m.l. estimator of $\rho_{jj'}$ is a solution of the cubic equation

$$\rho^3 - \frac{P_{jj'}}{N} \rho^2 + \left(\frac{S_j^2}{n} + \frac{S_{j'}^2}{n} - 1 \right) \rho - \frac{P_{jj'}}{n} = 0 \quad (5.9)$$

for which the second derivative of (5.7) is negative. Accordingly, for given values of $\hat{D}_{j,N}$ ($j=1, \dots, k$), we determine \hat{W}_{ij} ($i=1, \dots, N_j$ $j=1, \dots, k$), and $\hat{\eta}_j$ ($j=1, \dots, k$) and solve (5.9) by the Newton-Raphson method, starting with the initial solutions

$$\tilde{\rho}_j = 2P_{jj'} / (S_j^2 + S_{j'}^2), \quad j=1, \dots, k \quad (5.10)$$

In Table 5.1 we provide the initial estimates of the correlations (5.10) and the final estimates, which are the solutions of (5.9). The estimates of D_j are those given in Table 4.2. Two numbers are given in each cell of the matrix of final estimates of the correlations. The upper number is the correlation and the lower number is an estimate of its standard error, namely

$$S.E. \{ \hat{\rho}_{jj'} \} = (1 - \hat{\rho}_{jj'}^2) / \sqrt{N(1 + \hat{\rho}_{jj'}^2)} \quad (5.11)$$

Every estimate $\hat{\rho}_{jj'}$, which, in absolute value, is greater than $2 S.E. \{ \hat{\rho}_{jj'} \}$ is significant.

Table 5.1

Initial and final estimates of the
correlation parameters

INITIAL ESTIMATES OF CORRELATIONS

| | | | | | | | | | |
|-------|-------|--------|-------|-------|-------|-------|-------|--------|-------|
| 1.000 | 0.765 | 0.168 | 0.360 | 0.373 | 0.678 | 0.593 | 0.692 | 0.616 | 0.406 |
| 0.765 | 1.000 | 0.029 | 0.442 | 0.444 | 0.851 | 0.781 | 0.763 | 0.635 | 0.593 |
| 0.168 | 0.029 | 1.000 | 0.321 | 0.321 | 0.183 | 0.122 | 0.099 | -0.133 | 0.140 |
| 0.360 | 0.442 | 0.321 | 1.000 | 0.385 | 0.341 | 0.336 | 0.381 | 0.305 | 0.565 |
| 0.373 | 0.444 | 0.321 | 0.385 | 1.000 | 0.613 | 0.560 | 0.542 | 0.364 | 0.354 |
| 0.678 | 0.851 | 0.183 | 0.341 | 0.613 | 1.000 | 0.924 | 0.721 | 0.623 | 0.386 |
| 0.593 | 0.781 | 0.122 | 0.336 | 0.560 | 0.924 | 1.000 | 0.719 | 0.641 | 0.333 |
| 0.692 | 0.763 | 0.099 | 0.381 | 0.542 | 0.721 | 0.719 | 1.000 | 0.491 | 0.489 |
| 0.616 | 0.635 | -0.133 | 0.305 | 0.364 | 0.623 | 0.641 | 0.491 | 1.000 | 0.322 |
| 0.406 | 0.593 | 0.140 | 0.565 | 0.354 | 0.386 | 0.333 | 0.489 | 0.322 | 1.000 |

FINAL ESTIMATES OF CORRELATIONS

| | | | | | | | | | | |
|----|-------|-------|--------|-------|-------|-------|-------|-------|--------|-------|
| 1 | 1.000 | 0.760 | 0.163 | 0.355 | 0.348 | 0.676 | 0.587 | 0.694 | 0.583 | 0.394 |
| | 0.0 | 0.065 | 0.185 | 0.158 | 0.160 | 0.087 | 0.109 | 0.082 | 0.110 | 0.151 |
| 2 | 0.760 | 1.000 | 0.029 | 0.443 | 0.421 | 0.853 | 0.781 | 0.769 | 0.607 | 0.587 |
| | 0.065 | 0.0 | 0.192 | 0.141 | 0.146 | 0.040 | 0.059 | 0.062 | 0.104 | 0.109 |
| 3 | 0.163 | 0.029 | 1.000 | 0.319 | 0.299 | 0.183 | 0.120 | 0.101 | -0.120 | 0.136 |
| | 0.185 | 0.192 | 0.0 | 0.165 | 0.168 | 0.183 | 0.188 | 0.190 | 0.188 | 0.187 |
| 4 | 0.355 | 0.443 | 0.319 | 1.000 | 0.366 | 0.347 | 0.339 | 0.392 | 0.283 | 0.561 |
| | 0.158 | 0.141 | 0.165 | 0.0 | 0.157 | 0.160 | 0.161 | 0.152 | 0.170 | 0.115 |
| 5 | 0.348 | 0.421 | 0.299 | 0.366 | 1.000 | 0.596 | 0.538 | 0.527 | 0.322 | 0.330 |
| | 0.160 | 0.146 | 0.168 | 0.157 | 0.0 | 0.107 | 0.120 | 0.123 | 0.164 | 0.163 |
| 6 | 0.676 | 0.853 | 0.183 | 0.347 | 0.596 | 1.000 | 0.926 | 0.732 | 0.599 | 0.385 |
| | 0.087 | 0.040 | 0.183 | 0.160 | 0.107 | 0.0 | 0.020 | 0.072 | 0.106 | 0.153 |
| 7 | 0.587 | 0.781 | 0.120 | 0.339 | 0.538 | 0.926 | 1.000 | 0.726 | 0.615 | 0.328 |
| | 0.109 | 0.059 | 0.188 | 0.161 | 0.120 | 0.020 | 0.0 | 0.074 | 0.102 | 0.163 |
| 8 | 0.694 | 0.769 | 0.101 | 0.392 | 0.527 | 0.732 | 0.726 | 1.000 | 0.469 | 0.492 |
| | 0.082 | 0.062 | 0.190 | 0.152 | 0.123 | 0.072 | 0.074 | 0.0 | 0.136 | 0.131 |
| 9 | 0.583 | 0.607 | -0.120 | 0.283 | 0.322 | 0.599 | 0.615 | 0.469 | 1.000 | 0.294 |
| | 0.110 | 0.104 | 0.188 | 0.170 | 0.164 | 0.106 | 0.102 | 0.136 | 0.0 | 0.169 |
| 10 | 0.394 | 0.587 | 0.136 | 0.561 | 0.330 | 0.385 | 0.328 | 0.492 | 0.294 | 1.000 |
| | 0.151 | 0.109 | 0.187 | 0.115 | 0.163 | 0.153 | 0.163 | 0.131 | 0.169 | 0.0 |

6. Summary

In the current paper we present confidence intervals for the category scores as well as the correlations between the scores. The category scores help determine the weaknesses and the strengths of the units. It is important to obtain confidence intervals on these scores because before planning remedial training it is necessary to know whether or not the scores are meaningful. Thus high correlation between two categories may indicate that training to improve the performance in one can improve the performance in the other.

The results in Table 5.1 are intuitively explainable. We obtain high correlation between the categories evaluating the command's performance - Categories 1, 2, 6, 7, and 8. But Categories 3 (Communicating), 4 (Performing as Marines), 5 (Delivering supporting fire) and 10 (Supervising required actions of individual Marines) evaluate the performance of various specific groups and thus each of them cannot be expected to have high correlation with other categories.

A major finding - see Appendix A - is that, for any one battalion, the probability of "Yes" for a given requirement is the same for applicable requirements of the same category. This provides considerable justification for the use of 10 category scores as a measure of the overall capability of the battalion.

APPENDIX 1

We consider here the question of testing assumption A.2.

Typically, for a given evaluation, there are a certain number of "Yes's" among the applicable requirements. The question is how to test the hypothesis that the probability of a "Yes" in a given evaluation depends only on the category to which the requirement belongs. We have tested whether this hypothesis is supported by the data, in the following manner.

The data sets corresponding to the $N = 27$ different evaluations are arranged according to tasks within 17 different Military Performance Standards (MPS; see [7]). We therefore considered first the number of "Yes's" among the applicable requirements within each cell of MPS x category, for each evaluation. In Table A.1 we present these statistics, as an example, for EVAL 1. An exact test of the hypothesis that the probability of "Yes" is the same for all the MPS's within a category would be very difficult to perform, due to the small number of applicable requirements in each cell.

Let θ_{ikj} denote the probability of "Yes" in EVAL i , MPS k , Category j ($i=1, \dots, 27$; $k=1, \dots, 17$; $j=1, \dots, 10$). We consider the set of null hypotheses:

$$H_0^{(i,j)}: \theta_{ikj} = \theta_{ij} \text{ for all } k=1, \dots, 17 ;$$

$$\text{at each } i=1, \dots, 27 ; j=1, \dots, 10 .$$

In the spirit of the Bayesian approach of Section 3, we assume, a priori, that θ_{ikj} are independent random variables, having some prior distributions

$H_{kj}(\theta)$ ($k=1, \dots, 17$; $j=1, \dots, 10$). If the null hypotheses $H_0^{(i,j)}$ are true for all (i,j) , the following null hypotheses should also be true:
For each j , $j=1, \dots, 10$,

$$H_0^{(\cdot,j)}: \psi_{kj} = \psi_j \text{ for all } k=1, \dots, 17 ,$$

where

$$\psi_{kj} = \int_0^1 \theta_{ijk} dH_{kj}^{(\cdot,j)}(\theta_{ijk}) \quad (\text{A.1.1})$$

Table A.1

The number of satisfied (upper) and applicable (lower)
requirement by MPS and Category for Eval. 1

| MPS | CATEGORIES | | | | | | | | | |
|---------|------------|----|----|----|----|-----|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2.A.1 | 9 | 4 | 4 | 11 | 0 | 3 | 14 | 4 | 2 | 6 |
| | 11 | 9 | 4 | 19 | 0 | 3 | 18 | 13 | 3 | 10 |
| 2.A.2 | 8 | 0 | 20 | 0 | 7 | 10 | 15 | 9 | 9 | 3 |
| | 9 | 0 | 20 | 1 | 8 | 10 | 18 | 9 | 9 | 3 |
| 2.A.3 | 0 | 0 | 2 | 0 | 35 | 3 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 2 | 0 | 48 | 5 | 1 | 0 | 0 | 0 |
| 2.B.1 | 6 | 0 | 0 | 0 | 0 | 5 | 2 | 5 | 0 | 2 |
| | 6 | 0 | 0 | 0 | 0 | 5 | 3 | 5 | 1 | 2 |
| 2.B.2 | 7 | 5 | 1 | 2 | 0 | 8 | 3 | 7 | 2 | 1 |
| | 7 | 6 | 1 | 2 | 0 | 9 | 3 | 10 | 2 | 1 |
| 2.B.3 | 8 | 5 | 3 | 0 | 1 | 2 | 1 | 11 | 0 | 0 |
| | 8 | 5 | 3 | 0 | 1 | 3 | 1 | 14 | 0 | 0 |
| 2.B.4 | 9 | 3 | 1 | 0 | 2 | 9 | 5 | 12 | 2 | 1 |
| | 10 | 5 | 1 | 0 | 2 | 11 | 8 | 13 | 2 | 1 |
| 2.B.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.B.6 | 2 | 3 | 2 | 1 | 2 | 10 | 2 | 0 | 0 | 0 |
| | 2 | 4 | 2 | 1 | 2 | 12 | 3 | 0 | 0 | 0 |
| 2.B.7 | 3 | 2 | 0 | 0 | 0 | 6 | 0 | 1 | 2 | 0 |
| | 4 | 2 | 0 | 0 | 0 | 7 | 0 | 2 | 3 | 0 |
| 2.C.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.C.2 | 3 | 3 | 1 | 0 | 0 | 4 | 1 | 1 | 0 | 0 |
| | 3 | 3 | 1 | 0 | 0 | 4 | 1 | 4 | 0 | 0 |
| 2.C.3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.C.4 | 3 | 2 | 0 | 0 | 0 | 3 | 4 | 0 | 0 | 0 |
| | 3 | 2 | 0 | 0 | 0 | 3 | 4 | 0 | 0 | 0 |
| 2.D.1 | 2 | 3 | 6 | 0 | 8 | 34 | 0 | 0 | 4 | 0 |
| | 2 | 3 | 6 | 0 | 8 | 34 | 0 | 0 | 4 | 0 |
| 2.D.2 | 2 | 6 | 0 | 0 | 0 | 2 | 3 | 3 | 3 | 0 |
| | 2 | 6 | 0 | 0 | 0 | 2 | 3 | 3 | 8 | 0 |
| 2.D.3 | 0 | 8 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| | 0 | 9 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 |
| VOL. II | 62 | 44 | 40 | 14 | 55 | 100 | 50 | 53 | 24 | 13 |
| | 67 | 54 | 40 | 23 | 69 | 110 | 63 | 73 | 32 | 17 |

Let ψ_{kj}^* and M_{kj}^* ($k=1, \dots, 17$, $j=1, \dots, 10$) be the number of satisfied and applicable requirements in the k -th MPS and j -th category, summed over all the 27 evaluations. Let

$$X_{\cdot j}^* = \sum_{k=1}^{17} X_{kj}^* \quad (j=1, \dots, 10) \quad (A.1.2)$$

and

$$M_{\cdot j}^* = \sum_{k=1}^{17} M_{kj}^*$$

These statistics are given in Table A.2. We see in Table A.2 that the values of $M_{\cdot j}^*$ are large ones, for all $j=1, \dots, 10$; and that M_{kj}^* are generally not too small. We therefore consider, for each hypothesis $H_0^{(\cdot, j)}$, the chi-squared statistic

$$T_j^2 = \sum_{k=1}^{k_j^*} \frac{(X_{kj}^* - E_{kj}^*)^2}{E_{kj}^*} I_{kj}^* \quad , \quad (A.1.3)$$

where

$$E_{kj}^* = M_{kj}^* \frac{X_{\cdot j}^*}{M_{\cdot j}^*} \quad , \quad \begin{matrix} k=1, \dots, 17 \\ j=1, \dots, 10 \end{matrix} \quad (A.1.4)$$

$$I_{kj}^* = \begin{cases} 1 & , \text{ if } M_{kj}^* > 0 \\ 0 & , \text{ if } M_{kj}^* = 0 \end{cases} \quad (A.1.5)$$

and $k_j^* = \sum_{k=1}^{17} I_{kj}^*$. The estimates of the expected values of X_{kj}^* , under the null hypothesis $H_0^{(\cdot, j)}$, are E_{kj}^* . The large sample distribution of T_j^2 is like that of a chi-squared random variable with $v_j = k_j^* - 1$ degrees of freedom. In Table A.3 we provide the values of T_j^2 and their P -values

$$P_j = P \{X^2[v_j] \geq T_j^2 | T_j^2\} \quad , \quad j=1, \dots, 10 \quad (A.1.6)$$

Table A.2

The statistics $X_{.j}^*$ and $M_{.j}^*$

| | | | | | | | | | | |
|---------|------|-----|-----|-----|------|------|------|------|-----|-----|
| 1 | 214 | 158 | 66 | 366 | 0 | 65 | 351 | 216 | 64 | 219 |
| 1 | 245 | 199 | 93 | 472 | 0 | 70 | 413 | 283 | 75 | 258 |
| 2 | 212 | 0 | 416 | 18 | 162 | 218 | 367 | 204 | 207 | 70 |
| 2 | 232 | 0 | 447 | 26 | 182 | 239 | 440 | 228 | 225 | 75 |
| 3 | 0 | 0 | 36 | 0 | 993 | 77 | 21 | 0 | 0 | 0 |
| 3 | 0 | 0 | 49 | 0 | 1121 | 106 | 24 | 0 | 0 | 0 |
| 4 | 63 | 0 | 0 | 7 | 13 | 48 | 24 | 74 | 24 | 18 |
| 4 | 63 | 0 | 0 | 9 | 13 | 49 | 27 | 77 | 27 | 18 |
| 5 | 110 | 99 | 11 | 33 | 24 | 142 | 51 | 162 | 20 | 26 |
| 5 | 121 | 110 | 15 | 37 | 25 | 164 | 54 | 189 | 22 | 34 |
| 6 | 101 | 64 | 39 | 0 | 10 | 32 | 9 | 180 | 0 | 0 |
| 6 | 113 | 74 | 40 | 0 | 10 | 37 | 10 | 193 | 0 | 0 |
| 7 | 225 | 123 | 22 | 0 | 36 | 260 | 178 | 347 | 43 | 20 |
| 7 | 243 | 140 | 22 | 0 | 40 | 282 | 211 | 372 | 43 | 21 |
| 8 | 48 | 54 | 30 | 22 | 0 | 97 | 62 | 70 | 7 | 0 |
| 8 | 53 | 68 | 31 | 26 | 0 | 105 | 72 | 89 | 8 | 0 |
| 9 | 118 | 89 | 40 | 38 | 30 | 261 | 96 | 86 | 31 | 0 |
| 9 | 123 | 101 | 44 | 51 | 33 | 293 | 108 | 94 | 32 | 0 |
| 10 | 81 | 26 | 0 | 0 | 5 | 132 | 0 | 80 | 32 | 0 |
| 10 | 83 | 26 | 0 | 0 | 5 | 142 | 0 | 95 | 37 | 0 |
| 11 | 48 | 31 | 11 | 0 | 2 | 74 | 0 | 104 | 0 | 19 |
| 11 | 48 | 33 | 12 | 0 | 2 | 76 | 0 | 107 | 0 | 21 |
| 12 | 49 | 87 | 19 | 0 | 13 | 75 | 30 | 59 | 0 | 0 |
| 12 | 54 | 100 | 19 | 0 | 15 | 90 | 36 | 75 | 0 | 0 |
| 13 | 21 | 17 | 7 | 0 | 9 | 30 | 4 | 41 | 5 | 0 |
| 13 | 24 | 18 | 7 | 0 | 10 | 36 | 4 | 44 | 6 | 0 |
| 14 | 28 | 22 | 0 | 0 | 0 | 37 | 26 | 0 | 0 | 0 |
| 14 | 30 | 23 | 0 | 0 | 0 | 38 | 32 | 0 | 0 | 0 |
| 15 | 10 | 19 | 29 | 0 | 40 | 178 | 0 | 0 | 20 | 0 |
| 15 | 10 | 19 | 30 | 0 | 40 | 181 | 0 | 0 | 20 | 0 |
| 16 | 8 | 24 | 2 | 0 | 0 | 5 | 8 | 12 | 20 | 0 |
| 16 | 8 | 24 | 2 | 0 | 0 | 5 | 10 | 12 | 28 | 0 |
| 17 | 0 | 39 | 3 | 0 | 0 | 11 | 0 | 0 | 0 | 0 |
| 17 | 0 | 43 | 3 | 0 | 0 | 13 | 0 | 0 | 0 | 0 |
| TOTAL I | 1336 | 852 | 731 | 484 | 1337 | 1742 | 1227 | 1635 | 473 | 372 |
| TOTAL N | 1450 | 978 | 814 | 621 | 1496 | 1926 | 1441 | 1858 | 523 | 427 |

where $\chi^2[v_j]$ is a chi-squared random variable, independent of T_j^2 , with v_j degrees of freedom

Table A.3

The values of T_j^2 and P_j

| Cat. | T_j^2 | P_j | Cat. | T_j^2 | P_j |
|------|---------|--------|------|---------|--------|
| 1 | 2.155 | .99987 | 6 | 7.371 | .96543 |
| 2 | 3.811 | .99304 | 7 | 1.200 | .99996 |
| 3 | 7.326 | .88458 | 8 | 10.183 | .59991 |
| 4 | 1.089 | .95507 | 9 | 2.292 | .99357 |
| 5 | 1.112 | .99991 | 10 | 1.452 | .91854 |

We see in Table A.3 that each one of the ten null-hypotheses $H_0^{(\cdot,j)}$ is strongly accepted (very high P - values). Let P^* be the combined P - value of all the ten tests. According to the Bonferroni inequality [4],

$$P^* \geq 1 - \sum_{j=1}^{10} (1-P_j) \quad . \quad (A.1.7)$$

Thus, according to the values in Table A.3, $P^* \geq .30988$. Thus, we simultaneously accept all the ten null hypotheses, with a very high P^* value. Note that, if at least one of the null hypotheses had been rejected we should have concluded that assumption A.2 of Section 2 is not supported by the data. However, since all the null hypotheses have been strongly accepted, we conclude that the data is in accord with assumption A.2.

APPENDIX 2

We show in the present appendix that asymptotically, the correlation between $\hat{\theta}_{ij}$ and $\hat{\theta}_{ij'}$, is equivalent to $\rho_{jj'}$, for $j, j'=1, \dots, k$.

The inverse of (2.1) is

$$\hat{\theta}_{ij} = \sin^2(Y_{ij}/2), \quad \begin{matrix} i=1, \dots, N \\ j=1, \dots, k \end{matrix} \quad (\text{A.2.1})$$

where $Y_{ij} \sim N(\eta_{ij}, M_{ij}^{-1})$. Expand $\hat{\theta}_{ij}$ around η_{ij} to obtain

$$\begin{aligned} \hat{\theta}_{ij} &= \sin^2(\eta_{ij}/2) + (Y_{ij} - \eta_{ij}) \sin(\eta_{ij}/2) \cos(\eta_{ij}/2) \\ &\quad + O_P\left(\frac{1}{\sqrt{N}}\right), \end{aligned} \quad (\text{A.2.2})$$

where $O_P(\cdot)$ is an order-of-magnitude-in-probability symbol (see Zacks [5; pp. 208]).

Let $\hat{\theta}_{\sim i} = (\hat{\theta}_{i1}, \dots, \hat{\theta}_{ik})'$ and $\hat{\tau}_{\sim i} = \left(\sin^2\left(\frac{\eta_{i1}}{2}\right), \dots, \sin^2\left(\frac{\eta_{ik}}{2}\right)\right)'$.

Notice that $\sin\left(\frac{\eta}{2}\right) \cos\left(\frac{\eta}{2}\right) = \frac{1}{2} \sin(\eta)$. Let $(D)_i =$

$\text{diag}\{\sin(\eta_{i1}), \dots, \sin(\eta_{ik})\}$.

Then,

$$\hat{\theta}_{\sim i} = \hat{\tau}_{\sim i} + \frac{1}{2} (D)_i (Y_{\sim i} - \eta_{\sim i}) + O_P\left(\frac{1}{\sqrt{N}}\right), \quad (\text{A.2.3})$$

Thus, as $N \rightarrow \infty$, the asymptotic covariance matrix of $\hat{\theta}_{\sim i}$ is

$$\ddagger(\hat{\theta}_{\sim i}) = \frac{1}{4} (D)_i \ddagger_1 (D)_i, \quad (\text{A.2.4})$$

where the elements of \ddagger_1 are specified in (2.2). Accordingly, the asymptotic variance of $\hat{\theta}_{ij}$ is

$$AV \{\hat{\theta}_{ij}\} = \frac{1}{4} \sin^2(\eta_{ij})/M_{ij} \quad (A.2.5)$$

and the asymptotic covariance of $\hat{\theta}_{ij}$ and $\hat{\theta}_{ij'}$, $j \neq j'$, is

$$AC(\hat{\theta}_{ij}, \hat{\theta}_{ij'}) = \frac{1}{4} \rho_{jj'} \sin(\eta_{ij}) \sin(\eta_{ij'}) / \sqrt{M_{ij} M_{ij'}} \quad (A.2.6)$$

Hence, the asymptotic correlation between $\hat{\theta}_{ij}$ and $\hat{\theta}_{ij'}$ is $\rho_{jj'}$, for all $i=1, \dots, N$.

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